

A model of lepton masses from a warped extra dimension

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ABSTRACT: In order to explain the non-hierarchical neutrino mixing angles and the absence of lepton flavor violating processes in the context of warped extra dimensions one needs to introduce bulk flavor symmetries. We present a simple model of lepton masses and mixings in RS models based on the A_4 non-abelian discrete symmetry. The virtues of this choice are:

- (i) the natural appearance of the tri-bimaximal mixing pattern;
- (ii) the complete absence of tree-level flavor violations in the neutral sector;
- (iii) the absence of flavor gauge bosons;
- (iv) the hierarchies in the charged lepton masses are explained via wave-function overlaps.

We present the minimal field content and symmetry breaking pattern necessary to obtain a successful model of this type. The bounds from electroweak precision measurements allow the KK mass scale to be as low as ~ 3 TeV. Tree-level lepton flavor violation is absent in this model, while the loop induced $\mu \rightarrow e\gamma$ branching fraction is safely below the experimental bound.

KEYWORDS: Beyond Standard Model, Neutrino Physics, Field Theories in Higher Dimensions, Rare Decays.

Contents

1. Introduction	1
2. The setup	3
3. Higher order corrections and perturbativity bounds	8
4. Numerical scans and electroweak precision bounds	11
5. Constraints from lepton flavor violation	13
6. Conclusions	16
A. Summary of A_4 group theory	17
B. $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ at any order on IR brane	18
C. Review of gauge breaking via BCs	18

1. Introduction

Warped extra dimensions [1] provide a simple framework for fermion masses: exponential hierarchies are naturally generated due to the overlap of fermion and Higgs wave functions [2, 3], implementing the split fermion idea of [4]. In the simplest 5D “anarchic” approach [5, 6], where both the 5D bulk masses and the brane Yukawa couplings are assumed to be random, one generates a hierarchy both in the 4D standard model (SM) fermion masses and their mixing angles. This seems to fit the observed pattern of quark mixings and masses very well, since both the CKM matrix and quark masses show a hierarchical pattern. The lepton sector, however, is different: two of the observed neutrino mixing angles are close to maximal, rather than being hierarchical [7]. This suggests that one needs to radically change the implementation of fermion masses in warped extra dimensional models in order to achieve the correct lepton mixing pattern. Instead of a fully 5D anarchical approach, it calls for partial flavor symmetries, which will make sure that two of the neutrino mixing angles are not small, but close to maximal. The usual hierarchies can still be used to generate the hierarchies in charged lepton sector.

In fact, the necessary appearance of flavor symmetries is welcome for these models. Anarchic 5D flavor structure necessarily gives rise to flavor changing neutral currents (FCNC’s) in the quark sector, and to lepton flavor violation (LFV) in the lepton sector at tree-level [3, 5, 8, 6]. While these flavor violations are suppressed by the so-called RS-GIM

mechanism [8, 6], the KK mass scale still has to be as large as 20 TeV in order to safely suppress FCNC's in the quark sector [9]. In the lepton sector the anarchic approach [10] imposes a bound of order 10 TeV on the KK mass scale [11]. These bounds imply that the theory is out of reach for the LHC and unlikely to be useful for solving the (little) hierarchy problem. Reduction of these bounds require additional flavor symmetries [12, 13]. For the leptons, we have seen that such flavor symmetries are necessary to begin with, to protect the mixing angles from becoming hierarchical. The aim of this paper is to show that we can indeed use these flavor symmetries in the lepton sector to eliminate the LFV bounds on the model, and at the same time get a realistic pattern of masses and mixings. Recently, an alternative approach to lepton masses in RS was proposed in [14] (following the suggestion of [15]), which however does not readily explain the appearance of the large (non-hierarchical) neutrino mixing angles.

For our model, we pick the most popular global symmetry used in neutrino model building, an A_4 discrete symmetry [16, 17]. This, by the virtue of being discrete, has the added benefit that no additional gauge bosons have to be introduced, even if the symmetry is gauged. From the many studies of the implications of the A_4 symmetry in 4D models we expect that this symmetry can give the correct mixing structure. The mass hierarchies in the charged sector can still be generated as usual in RS models via fermion overlaps. The mass hierarchy in the neutrino sector is not that large, and can be readily incorporated by choosing $\mathcal{O}(1)$ factors in the neutrino mass matrix. Finally, since A_4 is a non-abelian discrete symmetry it has the potential of greatly reducing bounds from LFV. The reason for that is that by using a non-singlet representation under A_4 we can force the bulk wave functions of some of the left handed fields to be universal.

Indeed, we find that with appropriate choice of representations, the tri-bimaximal mixing pattern [18] characteristic of the A_4 symmetry can be reproduced by the leading order operators. Higher order terms can result in non-zero θ_{13} , while maintaining the predictions for θ_{12}, θ_{23} within the experimentally allowed range. Most importantly, our A_4 based model eliminates all tree-level sources for LFV. In fact, LFV is completely absent in the neutral current interactions, and shows up only through charged currents involving neutrino mixing. In the original RS neutrino mass model of [2] these loops turn out to be problematic [19], since the interaction of the heavy KK neutrinos with the SM fields is unsuppressed. In our model, by putting the charged leptons in the bulk (and peaked on the UV brane) these couplings are strongly suppressed and the bounds from loop induced decays are significantly reduced. The most important experimental bounds in the lepton sector are those from the electroweak precision (EWP) constraints. They are, however, quite mild, with KK masses of order 3 TeV generically allowed as in most other RS models.

The above arguments show that introducing the discrete non-abelian lepton flavor symmetry greatly improves over the generic RS lepton flavor models. Moreover, they are also improving the straight 4D implementations of A_4 neutrino models in several aspects. First, they explain the hierarchy in the charged lepton sector. Second, by putting one A_4 breaking VEV on the UV brane (breaking the group to Z_2), and the other on the IR (breaking it to Z_3) questions regarding vacuum alignment are eliminated. Finally, the appearance of the correct right handed neutrino mass scale (somewhat below M_{Pl} and

	SU(2) _L	SU(2) _R	U(1) _{B-L}	A ₄	Z ₂
Ψ _L	□	1	-1	3	-
Ψ _{e,μ,τ}	1	□	-1	1, 1', 1''	+
Ψ _ν	1	□	-1	3	-
H (IR)	□	□	0	1	+
ϕ' (IR)	1	1	0	3	-
ϕ (UV)	1	1	0	3	+

Table 1: Fields and their gauge and flavor charges.

M_{GUT}) can be explained by the partial compositeness of the right handed neutrino.

The paper is organized as follows: in section 2 we give the general setup, introduce the A_4 representations and calculate the mixing matrices at leading order. In section 3 we show the effects of higher dimensional operators on the mixing angles. In section 4 we present a numerical scan of the parameter space and discuss the electroweak precision bounds. In section 5 we show that LFV is completely absent at the tree-level in this model, and estimate the loop induced $\mu \rightarrow e\gamma$ branching fraction. We conclude in section 6.

2. The setup

We are assuming an AdS₅ bulk metric

$$ds^2 = \left(\frac{R}{z}\right)^2 (dx_\mu dx_\nu \eta^{\mu\nu} - dz^2), \tag{2.1}$$

with a UV brane at $z = R$ (R is also the AdS curvature scale) and an IR brane at $z = R'$. The magnitude of the scales is given by $R^{-1} \sim M_{Pl}$ and $R'^{-1} \sim 1 \text{ TeV}$. The electroweak gauge group is extended to an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry in the bulk to incorporate custodial symmetry [20]. This symmetry is reduced on the UV brane to the SM group $SU(2)_L \times U(1)_Y$, while it breaks down to $SU(2)_D \times U(1)_{B-L}$ on the IR brane.

The matter content is summarized in table 1. We assume that there is a separate doublet for every SM lepton, including the right handed neutrino: an $SU(2)_L$ doublet, Ψ_L , for every left handed doublet L , a separate $SU(2)_R$ doublet $\Psi_{e,\mu,\tau}$ for every right handed charged lepton, e, μ, τ , and a right handed doublet Ψ_ν for every right handed neutrino ν . A 5D fermion correspond to 2 Weyl fermions of opposite chirality in 4D:

$$\Psi = \begin{pmatrix} \chi \\ \bar{\psi} \end{pmatrix}. \tag{2.2}$$

In the KK decomposition, 4D chirality follows from the boundary conditions at the two end points of the extra dimension. The Lorentz structure forces χ and ψ to have opposite boundary conditions while a massless mode appears only for Neumann boundary condition at both ends. For a complete description of fermionic boundary conditions see [21]. The profile of the would-be zero mode is then entirely dictated by the 5D mass of Ψ . Conventionally, this mass is normalized as c/R and for $c > 1/2$ (resp. $c < -1/2$), a χ -zero mode

(resp. ψ -zero mode) is exponentially localized on the UV brane. In our setup, we assume that the boundary conditions (in the absence of the localized mass terms) are chosen as:

$$\Psi_L = \left(L \begin{bmatrix} +, + \end{bmatrix} \right) \Psi_{e,\mu,\tau} = \begin{pmatrix} \tilde{\nu}_{e,\mu,\tau} \begin{bmatrix} +, - \end{bmatrix} \\ e, \mu, \tau \begin{bmatrix} -, - \end{bmatrix} \end{pmatrix} \Psi_\nu = \begin{pmatrix} \nu \begin{bmatrix} -, - \end{bmatrix} \\ \tilde{l} \begin{bmatrix} +, - \end{bmatrix} \end{pmatrix} \quad (2.3)$$

where $[\pm]$ refers to a Neumann (Dirichlet) boundary condition on both branes for the χ component, while the $\bar{\psi}$ ones simply have the opposite conditions. Hence there is a left handed zero mode for the left handed doublets in Ψ_L , and a single right handed zero mode in $\Psi_{e,\mu,\tau}$ and Ψ_ν each.¹ These fields have bulk masses (in units of the AdS curvature) given by c_L, c_e and c_ν . For the general case these would be hermitian 3×3 flavor matrices. This is not the case here as we impose an $A_4 \times Z_2$ global symmetry in the bulk of the theory. In order to get the correct neutrino mass spectrum, we assign the three charged lepton doublets as well as the three right handed neutrinos to the 3 dimensional representation of A_4 . Thus for these fields there is just one common c -parameter each: c_L and c_ν . On the other hand, we assign the right handed charged leptons to the three inequivalent one dimensional representations of A_4 : $1+1'+1''$. Thus there are three separate c 's in this sector: c_e, c_μ and c_τ . The purpose of the Z_2 symmetry is to eliminate certain brane localized operators that would otherwise contribute to the mass matrices at leading order.

The symmetry breaking is achieved via brane localized scalars. Besides the SM Higgs, H , that is localized on the IR brane, we assume the following scalars to break the discrete symmetries: ϕ on the UV brane and ϕ' on the IR brane, both of which are in the 3 of A_4 . We assume that these two scalars develop VEVs in different directions: ϕ breaks A_4 to Z_2 , while ϕ' to Z_3 . This is achieved by the VEVs

$$\langle \phi' \rangle = (v', v', v'), \quad \langle \phi \rangle = (v, 0, 0), \quad (2.4)$$

in the basis where the generator corresponding to generator S of A_4 is diagonal (see appendix A for summary on A_4). Note, that once such a basis is chosen, these are the most general VEVs which preserve Z_3 and Z_2 subgroups of A_4 respectively, up to a trivial permutation of the basis.

We now write the most general Yukawa terms respecting both gauge and flavor symmetries (in addition one needs to write localized kinetic and potential terms for the localized scalars):

$$\begin{aligned} \mathcal{L}_{UV} &= -\frac{M}{2\Lambda} \psi_\nu \psi_\nu - x_\nu \frac{\phi}{2\Lambda} \psi_\nu \psi_\nu + \text{h.c.} + \dots, \\ \mathcal{L}_{IR} &= -\frac{y_\nu}{\Lambda'} \bar{\Psi}_L H \Psi_\nu - \frac{y_e}{\Lambda'^2} (\bar{\Psi}_L \phi') H \Psi_e - \frac{y_\mu}{\Lambda'^2} (\bar{\Psi}_L \phi'') H \Psi_\mu - \frac{y_\tau}{\Lambda'^2} (\bar{\Psi}_L \phi')' H \Psi_\tau + \text{h.c.} + \dots, \end{aligned} \quad (2.5)$$

where $\Psi^T = (\chi, \bar{\psi})$ and \dots stands for higher dimensional operators. We use the notation of [17] for writing A_4 -invariants: $(\cdot), (\cdot)'$ and $(\cdot)''$ denote the terms of 3×3 that transform as $1, 1'$ and $1''$ respectively, see appendix A.

¹It is well known that the $[-]$ boundary conditions (for a χ -type Weyl fermion) can lead to an extremely light KK-state for $c > 1/2$. Here we are safe from this problem as the right handed zero modes which satisfy this type of boundary conditions are localized close to the UV brane, i.e. $c < -1/2$.

Once the electroweak and A_4 symmetries are spontaneously broken, the boundary terms lead to boundary conditions mixing all fermions. Then, the spectrum is obtained by solving the bulk equation of motion in the presence of these mixed boundary conditions. The light modes, however, are quite insensitive to the boundary terms, and so they can be treated as a small perturbation. Hence to leading order the low energy spectrum may be obtained by using the zero mode wave functions. This defines the Zero Mode Approximation (ZMA) whose accuracy depends on how light the light masses are. As the largest mass is that of the τ , about 1 GeV, the ZMA turns out to be as accurate as $m_\tau R' \simeq 10^{-3}$ for zero mode masses.

To follow the conventional RS literature we introduce the RS flavor functions f and F ; these give the wave functions of the zero mode fermions on the IR and UV branes:

$$\chi_c(z) = \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c} f_c = \frac{1}{\sqrt{R}} \left(\frac{z}{R}\right)^{2-c} F_c \quad \text{and} \quad \psi_c(z) = \chi_{-c}(z), \quad (2.6)$$

with

$$f_c = \frac{\sqrt{1-2c}}{\sqrt{1-(R/R')^{1-2c}}}, \quad F_c = \frac{\sqrt{2c-1}}{\sqrt{1-(R/R')^{2c-1}}}. \quad (2.7)$$

The IR boundary terms of (2.6) lead to the following Dirac mass matrices for the zero mode charged leptons and neutrinos:

$$\mathcal{M}_{\mathbf{D}}^e = f_L \frac{v_H v'}{\sqrt{2} R' \Lambda'^2} \begin{pmatrix} y_e f_{-e} & y_\mu f_{-\mu} & y_\tau f_{-\tau} \\ y_e f_{-e} & \omega y_\mu f_{-\mu} & \omega^2 y_\tau f_{-\tau} \\ y_e f_{-e} & \omega^2 y_\mu f_{-\mu} & \omega y_\tau f_{-\tau} \end{pmatrix}, \quad (2.8)$$

$$\mathcal{M}_{\mathbf{D}}^\nu = y_\nu f_L f_{-\nu} \frac{v_H}{\sqrt{2} R' \Lambda'} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (2.9)$$

where we have introduced the shorthanded notation $f_i = f_{c_i}$ and $f_{-i} = f_{-c_i}$, ω is the cubic root of unity, $\omega = e^{2\pi i/3}$, and $v_H \sim 250$ GeV. The UV terms of (2.6) generate a Majorana mass matrix for right handed neutrinos of the form:

$$\mathcal{M}_{\mathbf{M}}^\nu = F_{-\nu}^2 R^{-1} \begin{pmatrix} \epsilon_s & 0 & 0 \\ 0 & \epsilon_s & \epsilon_t \\ 0 & \epsilon_t & \epsilon_s \end{pmatrix}. \quad (2.10)$$

with $\epsilon_s \equiv M/\Lambda$ and $\epsilon_t \equiv x_\nu v/\Lambda$.

Note, that while the mass hierarchy in the charged lepton sector is generated via the wave function overlaps in the usual way, the non-degeneracy in the neutrino sector is actually due to the two different kind of Majorana term allowed on the UV brane. As discuss below, the required neutrino mass hierarchy will actually require $\epsilon_s \sim \epsilon_t$. This is more naturally achieved if the singlet Majorana mass is actually also originating from an operator with a singlet scalar field VEV. This could for example be enforced by imposing an additional Z_3 global symmetry, and an additional scalar field ξ with VEV $\langle \xi \rangle = M$.

After integrating out the heavy right handed neutrinos, one ends up with a seesaw type Majorana mass matrix of the left handed neutrinos

$$\begin{aligned} \tilde{\mathcal{M}}_{\mathbf{M}}^{\nu} &\equiv -\mathcal{M}_{\mathbf{D}}^{\nu} \cdot (\mathcal{M}_{\mathbf{M}}^{\nu})^{-1} \cdot (\mathcal{M}_{\mathbf{D}}^{\nu})^T \\ &= -y_{\nu}^2 \frac{v_H^2 R}{2\Lambda'^2 R'^2} \frac{f_L^2 f_{-\nu}^2}{F_{-\nu}^2} \begin{pmatrix} 1/\epsilon_s & 0 & 0 \\ 0 & \epsilon_s/\Delta & -\epsilon_t/\Delta \\ 0 & -\epsilon_t/\Delta & \epsilon_s/\Delta \end{pmatrix}, \end{aligned} \quad (2.11)$$

with $\Delta \equiv \epsilon_s^2 - \epsilon_t^2$. The diagonalization procedure is the same as that of usual A_4 four-dimensional models. The charged lepton mass matrix becomes diagonal once the left-handed fields have been rotated as $L \rightarrow \mathbf{V} \cdot L$ with

$$\mathbf{V} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad (2.12)$$

a symmetric unitary matrix. We emphasize already here the most important properties of the A_4 mass matrices: the left handed rotation needed for diagonalizing the mass matrix is independent of the actual magnitudes of the masses, and the right handed rotation is the unit matrix (i.e. no right handed rotation is necessary). Therefore, the charged lepton masses are

$$\mathbf{V}^* \cdot \mathcal{M}_{\mathbf{D}}^e = f_L \frac{\sqrt{3} v_H v'}{\sqrt{2} R' \Lambda'^2} \begin{pmatrix} y_e f_{-e} & 0 & 0 \\ 0 & y_{\mu} f_{-\mu} & 0 \\ 0 & 0 & y_{\tau} f_{-\tau} \end{pmatrix}. \quad (2.13)$$

The charged lepton mass hierarchies are due to the hierarchies on $f_{-e, -\mu, -\tau}$ and the A_4 embedding of the right handed charged leptons allows for three different c 's which can be set to generate the physical charged lepton masses.

Next we move to the light neutrino sector. We work in the basis of diagonal charged leptons which is obtained by performing the rotation on the entire left-handed doublet with \mathbf{V} . Then, the light neutrino Majorana mass matrix is diagonalized by the Harrison-Perkins-Scott (HPS) matrix [18]

$$U_{\text{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \quad (2.14)$$

corresponding to $\theta_{13} = 0$, $\sin^2(2\theta_{12}) = 8/9$ and $\theta_{23} = \pi/4$. This tri-bimaximal mixing matrix is very close to the the best fit obtained from present oscillation data. The neutrino mass eigenstates are

$$U_{\text{HPS}}^T \cdot \mathbf{V}^* \cdot \tilde{\mathcal{M}}_{\mathbf{M}}^{\nu} \cdot \mathbf{V} \cdot U_{\text{HPS}} = -\tilde{m} \begin{pmatrix} \frac{1}{\epsilon_s + \epsilon_t} & 0 & 0 \\ 0 & \frac{1}{\epsilon_s} & 0 \\ 0 & 0 & \frac{1}{\epsilon_t - \epsilon_s} \end{pmatrix}, \quad (2.15)$$

where the overall mass scale is set by the combination

$$\tilde{m} \equiv y_{\nu}^2 \frac{v_H^2 R}{2\Lambda'^2 R'^2} \frac{f_L^2 f_{-\nu}^2}{F_{-\nu}^2}, \quad (2.16)$$

The neutrino mass-squared splittings are given by

$$\Delta m_{12}^2 \equiv |m_1|^2 - |m_2|^2 = \left| \frac{\tilde{m}}{\epsilon_s} \right|^2 \left[\frac{1}{(1+r)^2} - 1 \right], \quad (2.17)$$

$$\Delta m_{23}^2 \equiv |m_2|^2 - |m_3|^2 = \left| \frac{\tilde{m}}{\epsilon_s} \right|^2 \left[1 - \frac{1}{(1-r)^2} \right], \quad (2.18)$$

with $r \equiv \epsilon_t/\epsilon_s$, $|\Delta m_{12}^2| = \Delta m_{\text{sol}}^2$ and $|\Delta m_{23}^2| = \Delta m_{\text{atm}}^2$. Combining the last two relations we see that r solves the following algebraic equation:

$$r^3 - 3r - 2 \left(\frac{x-1}{x+1} \right) = 0, \quad (2.19)$$

where $x = \Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2$ for $|r| < 2$ or $x = -\Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2$ when $|r| > 2$. Finally, ϵ_s is found by inverting one of the two relations, for instance,

$$\epsilon_s = \frac{\tilde{m}}{\sqrt{\Delta m_{\text{atm}}^2}} \times \left(\left| 1 - \frac{1}{(1-r)^2} \right| \right)^{-1/2}. \quad (2.20)$$

When we impose the measured values [7] for the mass splittings

$$\Delta m_{\text{sol}}^2 \simeq 7.9 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{\text{atm}}^2 \simeq 2.6 \times 10^{-3} \text{ eV}^2, \quad (2.21)$$

we find four solutions for the neutrino mass spectrum corresponding to

$$r \approx \{0.79, 1.19, -2.01, -1.99\}. \quad (2.22)$$

The first (last) 2 solutions lead to a normal (inverted) hierarchy spectrum. Another general feature of A_4 is a prediction of the absolute neutrino mass scale, \tilde{m} . We find that the mass of the heaviest neutrino ends up being slightly above the atmospheric splitting.

We close this section by presenting a set of numerical values for the Lagrangian parameters which reproduce the mass spectra. The brane positions are $R^{-1} = 10^{19} \text{ GeV}$ and $R'^{-1} = 1.5 \text{ TeV}$, in order to keep the KK gauge bosons (with $m_{KK} = 3 - 4 \text{ TeV}$) in the reach of the LHC. The Higgs VEV turns out to be $v_H = 255.5 \text{ GeV}$, which is obtained after matching the bulk gauge couplings such that the weak boson masses as well as the fine-structure constant at the Z pole take their physical values: $m_W = 80.40 \text{ GeV}$, $m_Z = 91.19 \text{ GeV}$ and $\alpha_{\text{em}}^{-1}(m_Z) = 128$. As usual [22], we get a Higgs VEV slightly above the SM value due to the suppression of the W, Z wave functions on the IR brane and the additional contributions to the gauge boson masses from the wave function curvature terms.

The charged lepton masses are reproduced using the following choice of parameters: $c_L = 0.51$, $c_e = -0.75$, $c_\mu = -0.59$, $c_\tau = -0.51$, $y_e = 1.53$, $y_\mu = 1.55$, $y_\tau = 3.04$. Indeed, together with $\Lambda' = R'^{-1}$ and $v'R' = 0.1$, one gets $m_e = 0.511 \text{ MeV}$, $m_\mu = 106 \text{ MeV}$ and $m_\tau = 1.77 \text{ GeV}$. We show later on that the model defined with this set of parameters passes all leptonic electroweak precision bounds while perturbation theory remains under control up to $E = 3m_{KK}$. As we saw above, in the neutrino sector the solar and atmospheric mass splittings, eq. (2.21), reduce the ratio r to only four possible values, eq. (2.22). The corresponding mass spectra, as well as the Majorana masses on the UV brane (for $\Lambda = R^{-1}$, $x_\nu = 1$ and $c_\nu = -0.37$), are reported in table 2.

r	m_1	m_2	m_3	MR	vR
-2.01	53	54	18	-0.015	0.030
-1.99	55	54	18	-0.015	0.029
0.79	6.0	11	52	0.074	0.059
1.19	4.5	10	52	0.079	0.095

Table 2: Approximate numerical values of the neutrino masses and UV VEVs for the 4 possible solutions of $r \equiv \epsilon_t/\epsilon_s$. The masses are given in units of 10^{-3} eV.

3. Higher order corrections and perturbativity bounds

After spontaneous breaking of A_4 the lowest dimensional boundary terms of (2.6) generate the tri-bimaximal pattern for the mixing angles as well as the charged lepton and neutrino mass hierarchies. In order to consider this construction as really meaningful, it is necessary to study its stability under corrections from higher dimensional terms on the branes as well as radiative corrections. Another motivation for looking at deviations from tri-bimaximal mixings is the possibility to get a non-zero θ_{13} , in case it turns out to be non-vanishing experimentally.

We focus first on the UV brane and show that the effects of higher dimensional A_4 invariants lead to the same pattern for the Majorana mass matrix, except that some entries become complex. This is the only source of non-zero θ_{13} in this model. We start with writing down the most general higher order operators allowed on the UV boundary:

$$-\delta\mathcal{L}_{\text{UV}} = \sum_{n \geq 2} \lambda_n \frac{\phi^n}{\Lambda^n} \psi_\nu \psi_\nu + \text{h.c.} \quad (3.1)$$

with n insertions of ϕ . Now due to the Z_2 -preserving VEV $\langle \phi \rangle = (v, 0, 0)$, it is straightforward to show that ϕ^3 transforms as ϕ under A_4 . Thus all the higher order effects that cannot be reabsorbed into a redefinition of the lowest order parameters arise from one operator:

$$-\delta\mathcal{L}_{\text{UV}} = \lambda_2 \frac{\phi^2}{\Lambda^2} \psi_\nu \psi_\nu + \text{h.c.} \quad (3.2)$$

This term contains actually three independent A_4 invariants which lead to complex diagonal elements of the Majorana mass matrix and the pattern of (2.10) is to be replaced by:

$$\mathcal{M}_{\text{M}}^\nu = F_{-\nu}^2 R^{-1} \begin{pmatrix} \epsilon_s + \delta_1 & 0 & 0 \\ 0 & \epsilon_s + \delta_2 & \epsilon_t \\ 0 & \epsilon_t & \epsilon_s + \delta_2^* \end{pmatrix} \quad (3.3)$$

with $\delta_i \sim \mathcal{O}(v^2/\Lambda^2)$ and δ_1 real. The complex entries induce both a deviation of θ_{12} from its maximal value and a non-zero $\sin(\theta_{13})$ of $\mathcal{O}(\delta)$.

On the IR brane, we show that the higher order corrections can only affect θ_{12} , as long as Z_3 remains unbroken on this boundary. Again we start with the most general higher dimensional invariants which take the following form:

$$-\delta\mathcal{L}_{\text{IR}} = \sum_{i=e,\mu,\tau} \sum_{n \geq 2} \lambda'_{i,n} \bar{\Psi}_L \frac{\phi^n}{\Lambda^{m+1}} H \Psi_i + \sum_{n \geq 1} \kappa_n \bar{\Psi}_L \frac{\phi^n}{\Lambda^{m+1}} H \Psi_\nu + \text{h.c.} \quad (3.4)$$

Since $\langle \phi' \rangle = (v', v', v')$ is Z_3 symmetric, ϕ'^2 transforms as $1 + \phi'$. Then non-trivial corrections reduce to terms with only one ϕ' insertion:

$$-\delta\mathcal{L}_{\text{IR}} = \frac{\kappa_1}{\Lambda'^2} \bar{\Psi}_L \phi' H \Psi_\nu + \text{h.c.} \quad (3.5)$$

Since it is suppressed by only one power of (v'/Λ') compared to the lowest order terms, this operator may easily destabilize the HPS pattern. The extra Z_2 flavor symmetry is useful here, as we now discuss. We choose the A_4 triplets odd under this additional Z_2 , and then this operator is forbidden by the Z_2 symmetry. However, since $\phi'^2 = 1 + \phi'$, the next Z_2 even operator generates the same type of correction as the one linear in ϕ' but with a higher suppression factor. In that case (3.5) has to be replaced by

$$-\delta\mathcal{L}_{\text{IR}} = \frac{\kappa_2}{\Lambda'^3} \bar{\Psi}_L \phi'^2 H \Psi_\nu. \quad (3.6)$$

This operator corrects the Dirac neutrino mass matrix by introducing off diagonal elements of $\mathcal{O}(v'^2/\Lambda'^2)$. As this term contains three independent $A_4 \times Z_2$ invariants, the diagonal Dirac matrix of (2.9) becomes:

$$\mathcal{M}_{\mathbf{D}}^\nu = y_\nu f_L f_{-\nu} \frac{v_H}{\sqrt{2} R' \Lambda'} \begin{pmatrix} 1 + \epsilon_1 & \epsilon_2 & \epsilon_3 \\ \epsilon_3 & 1 + \epsilon_1 & \epsilon_2 \\ \epsilon_2 & \epsilon_3 & 1 + \epsilon_1 \end{pmatrix}, \quad (3.7)$$

with $\epsilon_i \sim \mathcal{O}(v'^2/\Lambda'^2)$. One can easily check (see appendix B) that only $\sin(\theta_{12})$ is affected and its deviation from the HPS prediction is of $\mathcal{O}(v'^2/\Lambda'^2)$.

There is another important feature we want to stress at that point, which is the fact that there is no correction to the charged lepton mass matrix. This means that the $\Psi_{e,\mu,\tau}$ fields need not be rotated again to get to the diagonal charged leptons basis even in the presence of these higher order terms. As we shall see in section 5, an immediate consequence is the absence of tree level LVF in this model, even when higher dimensional operators on are considered.

We close this section by presenting an NDA estimates for the allowed sizes of the IR brane localized operators. These bounds are important for estimating how large the deviations from the HPS mixing matrix could actually be. For this purpose, we write down again the most general Lagrangian on the IR brane

$$-\mathcal{L}_{\text{IR}} = \frac{y_\nu}{\Lambda'} \bar{\Psi}_L H \Psi_\nu + \frac{y_{e,\mu,\tau}}{\Lambda'^2} \bar{\Psi}_L H \phi' \Psi_{e,\mu,\tau} + \frac{\kappa_2}{2\Lambda'^3} \bar{\Psi}_L H \phi'^2 \Psi_\nu + \text{h.c.} \quad (3.8)$$

We require that the theory remains perturbative up to scale $E_N = N m_{KK}$, which corresponds to the first N^{th} KK modes of the theory being weakly coupled. We should require $N \geq 3$, such that at least the first few KK modes can be treated in perturbation theory. We can then systematically require that by the time this energy E_N is reached all loop corrections are still smaller than the lowest order terms. For example, for the first operator above there is a one loop correction to the Yukawa vertex itself (figure 1a), which implies

$$\frac{y_\nu^3}{16\pi^2} \frac{E_N^3}{\Lambda'^3} \leq y_\nu \frac{E_N}{\Lambda'}, \quad (3.9)$$

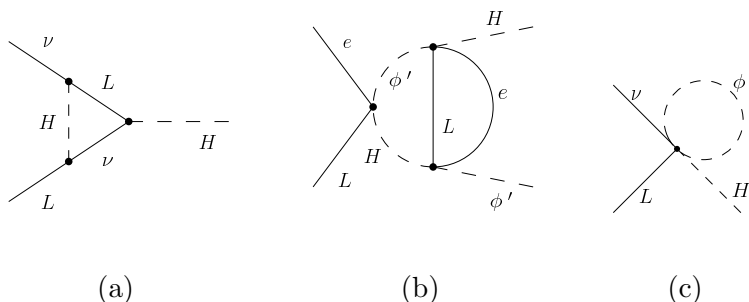


Figure 1: Loop diagrams correcting the operators localized on the IR brane which are relevant for NDA estimates.

where the linear running of the coupling has been taken into account. Recalling $m_{KK}R' \simeq 2$, this implies for $\Lambda' = R'^{-1}$ the perturbativity constraint

$$y_\nu \leq 2. \quad (3.10)$$

Similarly, for the second operator there is a two-loop correction to the tree-level operator (figure 1b), we get

$$\frac{y_e^3}{(16\pi^2)^2} \frac{E_N^6}{\Lambda'^6} \leq y_e \frac{E_N^2}{\Lambda'^2} \rightarrow y_e \leq 4 \quad (3.11)$$

again for $\Lambda' = R'^{-1}$. Finally, the third operator gives a one-loop correction to the first operator (figure 1c), which implies the relation

$$\frac{\kappa_2}{16\pi^2} \frac{E_N^3}{\Lambda'^3} \leq y_\nu \frac{E_N}{\Lambda'} \rightarrow \kappa_2 \leq 4y_\nu \leq 8. \quad (3.12)$$

We can then estimate that the higher dimensional terms corrections to $\sin(\theta_{12})$ are suppressed compared to the lowest order term by at least $4(v'R')^2$.

We now apply the same arguments on the UV brane where we have

$$-\mathcal{L}_{UV} = \frac{M}{2\Lambda} \psi_\nu \psi_\nu + x_\nu \frac{\phi}{2\Lambda} \psi_\nu \psi_\nu + \lambda_2 \frac{\phi^2}{4\Lambda^2} \psi_\nu \psi_\nu + \text{h.c.} \quad (3.13)$$

We require that the theory remains perturbative up to the natural scale on that boundary, namely until $E = R^{-1}$. First of all we focus on the last operator which contribute at one loop to its own vertex (figure 2d) which implies the usual constraint

$$\lambda_2 \leq 4\pi, \quad (3.14)$$

where we assumed $\Lambda = R^{-1}$. On the other hand, the second operator induces one-loop corrections to the mass M , the third operator, as well as its own vertex. From the diagrams of figures 2a, 2b and 2c we derive the following relations:

$$x_\nu \leq 4\pi\sqrt{\epsilon_s}, \quad x_\nu \leq \lambda_2^{1/4} \sqrt{4\pi} \leq (4\pi)^{3/4} \sim 7, \quad x_\nu \leq 4\pi. \quad (3.15)$$

We showed in the previous section that in order to reproduce the observed hierarchical neutrino mass splittings, one must have $\epsilon_s \sim \epsilon_t$ with $\epsilon_t = x_\nu(vR)$. Hence assuming a not

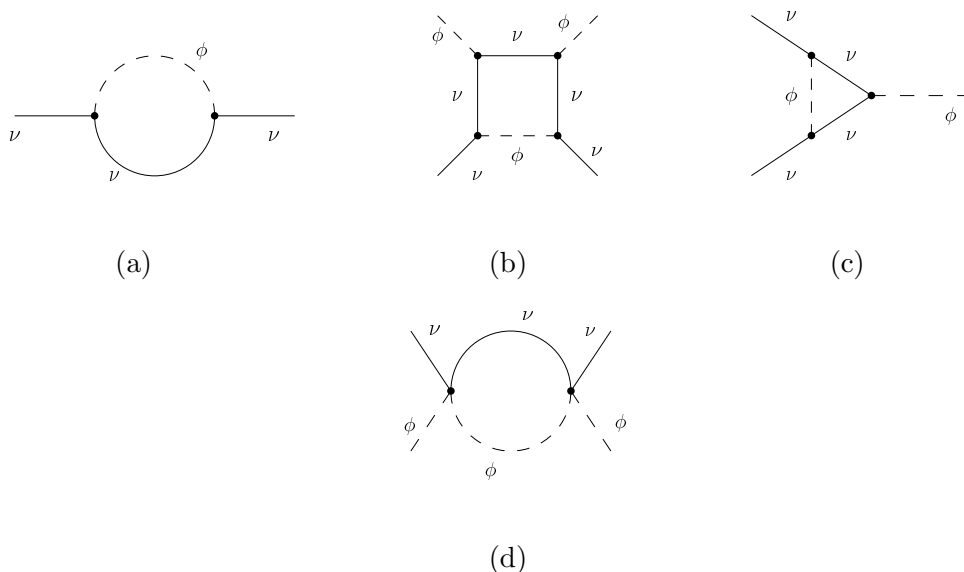


Figure 2: Loop diagrams illustrating the NDA estimates on the UV brane.

so small suppression factor $vR \sim 0.1$, such that θ_{13} is not dramatically tiny, the first of the above relations rewrites as $x_\nu \leq 16\pi^2(vR) \sim 15$. Thus we end up with the perturbative constraint

$$x_\nu \leq 7. \tag{3.16}$$

4. Numerical scans and electroweak precision bounds

One of the main constraints in models with new physics at the TeV scale are the electroweak precision measurements (EWPM). For RS models with custodial symmetry the generic bound on the KK scale is about $m_{KK} \geq 3\text{TeV}$, mostly from the contribution to the S-parameter [20]. Here we will check that the lepton sector of our model indeed passes these tests for KK scales of order 3 TeV.^{2,3}

The simplest way of checking the electroweak precision constraints in an RS model with bulk fields is to canonically normalize the SM gauge fields, and to determine the parameters g_5 , g'_5 and v_H by requiring that the measured values of M_W, M_Z and the electromagnetic coupling e are reproduced. This choice is somewhat unconventional, since M_W is less accurately measured than G_F , however it simplifies the matching of the 5D parameters to the observables significantly. In this scheme all corrections to electroweak precision observables will be contained in the fermion-gauge boson vertices, which can be simply calculated by wave function overlaps, and compared to the SM predictions in terms

²It is possible to cancel the S-parameter by tuning all LH bulk fermion masses to be ~ 0.5 [23], as it is necessary in higgsless models [24]. In this case however one does not get any mass hierarchies and those need to be introduced by hand as in [12].

³See also [25] for an attempt to reproduce the lepton masses and mixings with a lower KK mass scale.

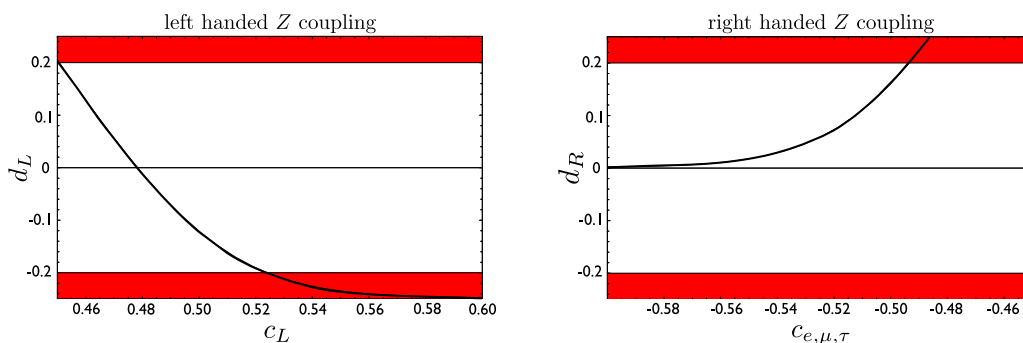


Figure 3: Deviation from SM Z couplings of l_L and l_R as function of c 's. $d_L = (g_Z^L - g_{Z,SM}^L)/g_{Z,SM}^L$ and $d_R = (g_Z^R - g_{Z,SM}^R)/g_{Z,SM}^R$ are plotted in percent units. We took $R'^{-1} = 1.5$ TeV and used $m_W = 80.403$ GeV, $m_Z = 91.1876$ GeV and $e = e(\mu = m_Z) = \sqrt{4\pi/128}$ as physical input observables. The red regions are excluded by EWPM in the leptonic sector.

of the above input parameters, which for the lepton- Z -couplings are

$$g_{Z,SM}^L = e \left(\frac{1}{2} - \frac{m_W^2}{m_Z^2} \right) \frac{m_Z}{m_W \sqrt{1 - \frac{m_W^2}{m_Z^2}}}, \quad (4.1)$$

$$g_{Z,SM}^R = e \frac{m_Z}{m_W} \sqrt{1 - \frac{m_W^2}{m_Z^2}}. \quad (4.2)$$

In a warped extra dimension the couplings of the left and right handed charged leptons are:

$$g_Z^L \simeq \frac{1}{2} \int_R^{R'} dz \left(\frac{R}{z} \right)^4 \left[[g_{5L} a^{L,3}(z, m_Z) + \tilde{g}_5 a^X(z, m_Z)] \chi_{c_L}(z)^2 \right] \quad (4.3)$$

$$g_Z^{R,i} \simeq \frac{1}{2} \int_R^{R'} dz \left(\frac{R}{z} \right)^4 \left[[g_{5R} a^{R,3}(z, m_Z) + \tilde{g}_5 a^X(z, m_Z)] \psi_{c_i}(z)^2 \right]. \quad (4.4)$$

with $a(z, m_Z)$ the wave functions of the three neutral gauge fields containing the physical Z , while the various g_5 's denote the bulk gauge couplings.⁴ The origin of the deviations from the SM predictions is the non-flatness of the Z wave function. If the Z boson was massless, its wave function would be flat and the lepton couplings would become universal thanks to the orthonormality of their wave functions. However as soon as the A_4 symmetry is imposed, the left handed lepton couplings remain flavor blind. In the ZMA the Z coupling depends only on the bulk mass parameter c_L . As usual we assume that the SM leptons are localized on the UV brane, $c_L > 1/2$, $c_{e,\mu,\tau} < -1/2$. Since the τ is the heaviest lepton, it has to be localized closest to the IR brane, so it will be most sensitive to the non-flatness of the Z close to the IR brane, and so one expect the biggest deviations in the τ couplings.

⁴For completeness we review in appendix C how this quantities are calculated in terms of physical observables.

The couplings of the charged leptons have been measured very precisely at the LEP experiment [7]. Here we will require that all lepton couplings are within 0.2% of the SM prediction. The plots of figure 3 show the deviation of the charged lepton couplings to the Z from their SM values as a function of the c 's. We see that, while the $c_{e,\mu,\tau}$ can be as close to $-1/2$ as required to reproduce the charged fermion hierarchy, c_L cannot depart too much from $1/2$ to remain within the experimental bound. The fact that c_L is preferred to be close to $1/2$ may be surprising at first, but we remind the reader that these vertex corrections also include the S-parameter. This is actually a welcome fact, since in order to keep the τ Yukawa coupling perturbative we have to take c_L close to $1/2$ anyway. Thus we conclude from figure 3 that with a 3 TeV KK mass scale the electroweak precision constraints in the lepton sector are safely satisfied.

Next we want to scan over the model's parameters and show explicitly that the neutrino mixing angles, which deviate from the HPS pattern in the presence of higher order operators, are within the allowed range of the existing results of the neutrino experiments. Once the usual RS solution to the hierarchy problem is imposed, we still have 12 free parameters in our setup: 5 c 's, 5 Yukawas and 2 VEVs relevant for lepton masses v, v' , while adding higher order operators brings 6 more coupling constants. We use the measured lepton masses and the best fit neutrino mass splittings to fix 5 of the lowest order parameters, which leaves still a lot of freedom to explore. Therefore we add some mild assumptions in order to simplify the parameter space and to try to only focus on the main predictions without having to go into the details of the structure of the higher dimensional operators. First of all we impose $c_\tau = -c_L$ and keep c_L as a free parameter. We also take the Yukawas on the IR brane to saturate the perturbative bounds: $y_{e,\mu,\tau} = 2y_\nu = 4$. Thus imposing $m_\tau = 1.77$ GeV, $m_e = 0.511$ MeV and $m_\mu = 106$ MeV in turn fixes v' and $c_{e,\mu}$ as functions of c_L . Hence all IR brane effects will be encoded in one parameter c_L , which is constrained by the EWPM as discussed above. On the UV brane the solar and atmospheric splittings fix the ratio M/v and the overall neutrino mass scale. Then taking $x_\nu = 1$ we end up with $v = v(c_\nu)$ as the only free parameter on this boundary. Furthermore, $(vR)^2$ controls the size of θ_{13} generated through higher order corrections. Finally we present in figure 4 some contours of the mixing angles in the plane (c_L, c_ν) when one among the possible combinations of higher order invariants are included on both branes. We considered separately the effect on θ_{12} from the IR brane and the predictions for all angles from the UV brane. In order to demonstrate the robustness of the tri-bimaximal pattern under higher order corrections in this model we have selected the worst case where these operators saturate their perturbative limits, namely for $\lambda_2 = 4\pi$ and $\kappa_2 = 8$. We conclude from figure 4 that, even when the deviations from HPS angles are the largest possible, there is still a viable region satisfying the constraints $\sin^2(2\theta_{13}) < 0.19$ (90CL), $\sin^2(2\theta_{23}) > 0.92$ (90CL) and $0.73 \leq \sin^2(2\theta_{12}) \leq 0.95$ (3σ) [7, 26]. Obviously the smaller the higher order terms are the closer one would get to the HPS pattern.

5. Constraints from lepton flavor violation

Flavor models usually predict new sources of flavor violations, and so are only viable when the flavor scale is pushed to very high values. The flavor scale for quarks in the usual

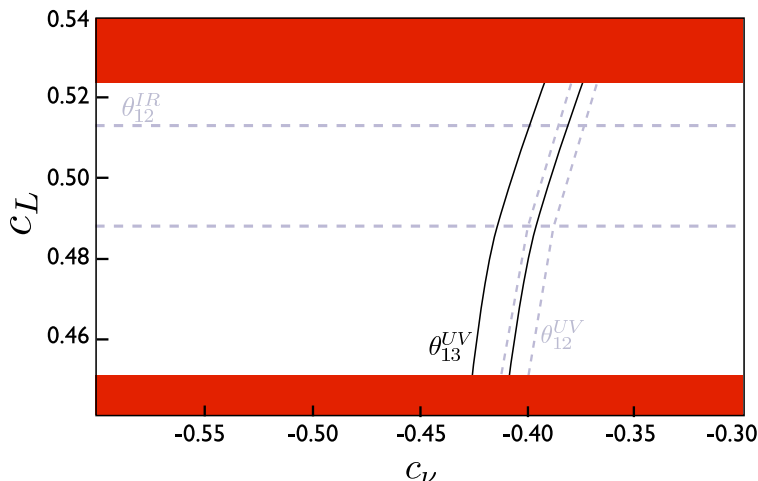


Figure 4: Scan of the parameter space reduced to (c_L, c_ν) as motivated in the text. The red regions are excluded by electroweak precision constraint on the Z coupling. We then show within this region some contours of the mixing angles delineating the largest values we typically obtain in the presence of higher dimensional corrections. The two horizontal contours are for the following values of $\sin^2(2\theta_{12}) = 0.90, 0.95$ (from bottom to top) where only the IR higher dimensional operator is added with $\epsilon_1 = \epsilon_2 = 0$ and $\epsilon_3 = 8(v'R')^2$. The oblique lines are contours of $\sin^2(2\theta_{13}) = 0.01, 0.19$ (from left to right) and contours of $\sin^2(2\theta_{12}) = 0.90, 0.95$ (from left to right) generated by the higher order Majorana mass on the UV brane, for $\delta_1 = \delta_2 = 0$ and $\delta_3 = 4\pi(vR)^2$.

anarchic RS flavor models is around 20 TeV [9]. Thus it is very important to understand what the possible sources of lepton flavor violation (LFV) are in this model of lepton masses. Generically there are two types of LFV sources: tree-level LFV via the exchange of heavy neutral particles (like Z') or off-diagonal Z couplings, or loop induced rare decays via charged current interactions.

Tree-level LFV operators have been considered in [11] (see also [27]), and it was found that the lepton flavor scale is at least 5 TeV, and larger for smaller brane Yukawa couplings. We will first show here that the structure of the A_4 symmetry used here is such that all tree-level sources of lepton flavor violation are absent in this theory. This can be seen in the following way. By the choice of the A_4 representation the wave functions of the left handed SM fermions are flavor universal (since they have the same c_L). So the only source of flavor violations in the charged lepton sector is the choice of different $c_{e,\mu,\tau}$ necessary for the mass hierarchies, and the brane Yukawa matrix (2.8). The couplings to the neutral bulk fields like the KK tower of the Z are controlled by the $c_{e,\mu,\tau}$, and will be non-universal. Thus any RH rotation to diagonalize the brane Yukawa matrix would induce tree-level LFV's. However, we have seen that one of the magic properties of the A_4 models is that the charged lepton mass matrix is diagonalized by a single left handed rotation, and no right handed rotation is necessary to diagonalize the mass matrix, as described in (2.13). The left-handed rotation is harmless, since the bulk wave functions are universal in the LH sector, while in the dangerous RH sector there is no rotation necessary. This implies that there is a basis where the kinetic terms and the mass terms for the charged leptons

are simultaneously diagonal, and so as a consequence there is no tree-level LFV in this model. The lepton embedding in A_4 provided us with the necessary conditions to ensure the absence of LFV, namely universal left handed c 's *and* the absence of redefinition of the right handed fields. Moreover we want to stress that this result remains unchanged once higher dimensional brane operators are considered as they were shown not to affect the rotation matrices of the charged leptons.

Thus all lepton flavor violation arises from charged current interactions. In the SM loops involving neutrinos give extremely small contributions to rare decays, however in the presence of heavy KK neutrinos this is no longer the case. For example in the case of the original neutrino mass model of [2] the large splittings of the heavy neutrinos, together with their unsuppressed couplings to the SM fields yield a large loop-induced $\mu \rightarrow e\gamma$ rate. In fact, Kitano [19] showed that a bound of $m_{KK} > 25$ TeV applies in this case. In our case however there is a generic softening of this bound, due to the fact that the SM fermions are now localized close to the UV brane, and therefore the charged current interactions with the KK neutrinos will be suppressed. This will happen generically in any model with bulk fermions. For the particular case of the A_4 model the situation is even better: since the second Yukawa coupling of the charged leptons involving the right handed neutrinos is uniform due to the A_4 representations, the interactions with all higgses (neutral or charged) will be diagonalized in the same basis where the masses are diagonalized. So one only needs to consider the exchange of charged W's and their KK towers, together with KK neutrinos. In fact, just as in the SM the loop induced contribution here will be finite. The reason for that is that the allowed additional brane localized counter term is of the form $LH\phi'\sigma^{\mu\nu}e_RF_{\mu\nu}$, and this will be diagonalized once the charged lepton mass matrix is diagonal. This is again a specific property of the structure of the A_4 invariants.

Next we will give a rough estimate of the KK mass bound from these processes. We will focus on loop induced $\mu \rightarrow e\gamma$ decays via to exchange of a W-bosons and KK neutrinos. The branching fraction from the exchange of a heavy neutrino and a W was calculated by Cheng and Li [28] and is given by (assuming the coupling to the W given by the usual SM gauge coupling g):

$$B(\mu \rightarrow e\gamma) = \frac{3\alpha}{8\pi} \left| \sum_i U_{\mu i}^* U_{ei} F\left(\frac{m_i^2}{M_W^2}\right) \right|^2 \tag{5.1}$$

where the sum over i indicates the sum over a generation of KK fermions, U is the PMNS mixing matrix between the SM charged leptons and a generation of KK neutrinos, and the function F is given by

$$F(z) = \frac{1}{6(1-z)^4} (10 - 43z + 78z^2 - 49z^3 + 18z^3 \log z + 4z^4). \tag{5.2}$$

First we specify to the case of the exchange of a SM W and a KK mode neutrino. For $z \gg 1$ the approximate expression is $F(z) \approx \frac{2}{3} + 3\frac{\log z}{z}$. Also, the coupling between the zero mode SM fermions, a KK neutrino and the zero mode W is suppressed at least by one factor of f_L , so there is an overall f_L^4 appearing in the rate. This is the main difference compared to the analysis of [19]: there all SM fermions were localized on the TeV brane,

so the interaction with a KK neutrino was unsuppressed. The leading term drops out due to the unitarity of U , and so we are left with the approximation

$$B(\mu \rightarrow e\gamma) < f_L^4 \frac{54\alpha}{\pi} \frac{m_W^4}{m_{KK}^4} \frac{\delta m_{KK}^2}{m_{KK}^2} \log^2 \frac{m_{KK}}{m_W}, \quad (5.3)$$

where δm_{KK} is the characteristic splitting among the KK modes in a given generation, given by $\frac{\delta m_{KK}^2}{m_{KK}^2} \approx \frac{y^2 v_H^2}{2m_{KK}^2}$. We find, that even for Yukawa coupling close to the perturbative limit $y \sim 3$, and c_L close to the composite case $c_L = 0.5$ the branching ratio is two orders of magnitude below the experimental bound of 10^{-11} for a KK mass scale of 3 TeV.

A slightly bigger contribution is obtained using the diagram where in addition to the KK neutrino one exchanges a KK W. In this case the gauge coupling could be as large as $gf_L \sqrt{\log R'/R}$. In addition the branching ratio (5.1) is suppressed by $(m_W/m_{KK})^4$ since the decay rate $\Gamma(\mu \rightarrow e\gamma)$ scales as the fourth inverse power of the exchanged gauge boson mass. One needs to take the function $F(z)$ at values $z \sim 1$ for which it is approximately given by $F(z \sim 1) \sim \frac{17}{12} + \frac{3}{20}(1-z)$. The universal piece drops out again due to the unitarity of U and we get an upper bound to the resulting branching fraction of order

$$B(\mu \rightarrow e\gamma) < f_L^4 \frac{27\alpha}{800\pi} \frac{m_W^4}{m_{KK}^4} \frac{\delta m_{KK}^2}{m_{KK}^2} \log^2 \frac{R'}{R}, \quad (5.4)$$

which again numerically is smaller than 10^{-13} for a 3 TeV KK mass. Clearly since the numerical contributions turn out to be not that far from experimentally interesting region it would be very interesting to perform a more detailed calculation of the $\mu \rightarrow e\gamma$ branching fraction in this model, including complete sums over KK towers (and also in general RS models with bulk fermions and Majorana neutrinos).

6. Conclusions

Warped extra dimensions provide a successful framework for flavor models: hierarchies in the masses and the mixing angles are naturally generated. Since neutrinos do not show hierarchies in the mixing angles, and only a mild hierarchy in the mass spectrum, one should introduce additional symmetries in the lepton sector. In this paper we have augmented the lepton sector of the RS model with the most successful and economical global symmetry used for neutrino mass models, the discrete non-abelian group A_4 . With appropriate assignments of the A_4 representations we can naturally achieve a successful lepton mixing pattern, while the charged lepton mass hierarchy is implemented as usual in RS models via wave function overlap. The A_4 symmetry also eliminates all tree-level sources of LFV in the neutral current sector, and so significantly lowers the bound on the KK mass scale. LFV appears only through charged current loops, and we estimated that the rate of $\mu \rightarrow e\gamma$ is safely below the current experimental bound. So the most significant bounds on this model come from the EWP measurements, and as usual with KK mass scales of order 3 TeV these corrections will be also safely within the experimental bounds.

Acknowledgments

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A. Summary of A_4 group theory

A_4 is an $SO(3)$ subgroup which leaves the tetrahedron invariant. It has 12 elements, two generators (S, T) connecting all of them, and four irreducible representations: one three-dimensional (3) and three one-dimensional ($1, 1'$ and $1''$, with $(1')^* = 1''$). Their products decompose as:

$$\begin{aligned}
 1' \times 1'' &= 1, & 1 \times 3 &= 3 \\
 1' \times 1' &= 1'', & 1' \times 3 &= 3 \\
 1'' \times 1'' &= 1', & 1'' \times 3 &= 3 \\
 3_x \times 3_y &= 3_1 + 3_2 + 1 + 1' + 1'' & & \quad (A.1)
 \end{aligned}$$

where for the last line, with $3_x = (x_1, x_2, x_3)$, $3_y = (y_1, y_2, y_3)$ and working in a basis where the three-dimensional representation of S is diagonal:

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (A.2)$$

one has:

$$1 = x_1 y_1 + x_2 y_2 + x_3 y_3 \quad (A.3)$$

$$1' = x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3 \quad (A.4)$$

$$1'' = x_1 y_1 + \omega x_2 y_2 + \omega^2 x_3 y_3 \quad (A.5)$$

$$3_1 = (x_2 y_3, x_3 y_1, x_1 y_2) \quad (A.6)$$

$$3_2 = (x_3 y_2, x_1 y_3, x_2 y_1) \quad (A.7)$$

with $\omega = e^{2\pi i/3}$ the cubic root of unity, satisfying:

$$1 + \omega + \omega^2 = 0, \quad \omega^* = \omega^2. \quad (A.8)$$

Note also that one has: $3 \times 1' = 3 \sim u(x_1, \omega x_2, \omega^2 x_3)$, where $u \sim 1'$. The same holds for $3 \times 1''$ with $\omega \rightarrow \omega^2$.

B. $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ at any order on IR brane

We explicitly show in this section that the higher dimensional operators on the IR brane only affect θ_{12} . We recall that the Dirac mass matrix on the IR brane in presence of higher orders is of the form:

$$\mathcal{M}_{\mathbf{D}}^{\nu} = \begin{pmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{pmatrix} \quad (\text{B.1})$$

while at lowest order the Majorana mass matrix on the UV is:

$$\mathcal{M}_{\mathbf{M}}^{\nu} = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}. \quad (\text{B.2})$$

After the seesaw the Majorana mass matrix for the left-handed neutrinos (in the basis of diagonal charged leptons) shows the following pattern:

$$\tilde{\mathcal{M}}_{\nu} = \begin{pmatrix} b & c & c^* \\ c & g & f \\ c^* & f & g^* \end{pmatrix}. \quad (\text{B.3})$$

Note that even for real input parameters this matrix has complex entries due to $\omega = e^{2i\pi/3}$ factors that do not cancel out when the left-handed doublet is rotated with \mathbf{V} . If c and g were real, the Majorana mass matrix would be diagonalized with $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. In general it is a 3×3 complex symmetric matrix which thus contains 12 independent parameters. They are the 3 real eigenvalues, the 3 mixing angles and 6 phases. Moreover one can always redefine the neutrino fields to absorb 3 of them, leaving only 2 Majorana phases and a CKM one. Thus with the redefinition $\nu_i \rightarrow e^{i\phi_i} \nu_i$ the mass matrix becomes:

$$\tilde{\mathcal{M}}_{\nu} = \begin{pmatrix} b e^{2i\phi_1} & |c| e^{i(\phi_c + \phi_1 + \phi_2)} & |c| e^{i(\phi_1 + \phi_2 - \phi_c)} \\ |c| e^{i(\phi_c + \phi_1 + \phi_2)} & |g| e^{i(2\phi_2 + \phi_g)} & f e^{i(\phi_2 + \phi_3)} \\ |c| e^{i(\phi_1 + \phi_2 - \phi_c)} & f e^{i(\phi_2 + \phi_3)} & |g| e^{i(2\phi_3 - \phi_g)} \end{pmatrix}. \quad (\text{B.4})$$

It is now not difficult to see that this matrix can be made real with $\phi_1 = 0$, $\phi_2 = -\phi_3$ and $\phi_3 = \phi_c$ provided that the relation $2\phi_c = \phi_g$ holds. Although the expressions of these phases in terms of the input parameters are quite cumbersome, we checked that the latter relation is satisfied in our model. Therefore we conclude that the most general higher dimensional corrections on the IR brane can only modify θ_{12} from its HPS value.

C. Review of gauge breaking via BCs

We shortly review here how the bulk $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry is broken in this setup. The main goal is to define how to get the gauge boson profiles as we will need them to compute the W and Z couplings to the standard model fermions. We note A^L , A^R and A^X as well as g_{5L} , g_{5R} and \tilde{g}_5 the gauge fields and coupling constants associated with

$SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively. On the UV brane, the gauge symmetry breaks down to the SM gauge group $SU(2)_L \times U(1)_Y$ from Dirichlet BC⁵ for the $SU(2)_R$ fields:

$$z = R : \begin{cases} A_\mu^{R,\pm} = \partial_z A_\mu^{L,\pm} = 0 \\ \partial_z A_\mu^{L,3} = 0 \\ \tilde{g}_5 A_\mu^X - g_{5R} A_\mu^{R,3} = 0 \\ \partial_z (g_{5R} A_\mu^X + \tilde{g}_5 A_\mu^{R,3}) = 0 \end{cases} \quad (C.1)$$

where $A^\pm \equiv (A^1 \mp iA^2)/\sqrt{2}$. The chiral $SU(2)$'s are broken to the vectorial subgroup on the IR brane by the finite VEV of a Higgs bidoublet $\langle h \rangle = \text{diag}(v_H, v_H)/2$, and the resulting (mixed) BCs are:

$$z = R' : \begin{cases} \partial_z (g_{5L} A_\mu^{L,a} - g_{5R} A_\mu^{R,a}) + \mathcal{V} (g_{5L} A_\mu^{L,a} - g_{5R} A_\mu^{R,a}) = 0 \\ \partial_z (g_{5R} A_\mu^{L,a} + g_{5L} A_\mu^{R,a}) = 0 \\ \partial_z A_\mu^X = 0 \end{cases} \quad (C.2)$$

where $\mathcal{V} = (R'/R)(g_{5L}^2 + g_{5R}^2)v_H^2/4$. The fifth components of the gauge fields have opposite BC. Obviously, these BCs allow for a massless (flat) mode which is nothing else but the photon field associated with the unbroken $U(1)$ of the (compactified) effective theory, and the KK decomposition is of the form:

$$A_\mu^{L,R,\pm}(x, z) = a^{L,R}(z, m_W) W_\mu^\pm(x) + \dots \quad (C.3)$$

$$A_\mu^{L,R,3}(x, z) = \frac{\tilde{g}_5}{g_{5L,R}} a_0 \gamma_\mu(x) + a^{L,R,3}(z, m_Z) Z_\mu(x) + \dots \quad (C.4)$$

$$A_\mu^X(x, z) = a_0 \gamma_\mu(x) + a^X(z, m_Z) Z_\mu(x) + \dots \quad (C.5)$$

where the \dots stand for heavier KK resonances, and the wave functions are given by $a(z, m) = z(AJ_1(mz) + BY_1(mz))$.

Yet remains the definition of the 5D gauge couplings. For this we have to match the latter on the 4D SM couplings. The fact that there is no $SU(2)_L \times U(1)_Y$ symmetry in the effective 4D action makes the definition of the SM couplings somehow arbitrary. As matching conditions, we choose to recover the measured values of m_W , m_Z and the electric charge e . After having fixed the values of R , R' and the ratio of the left/right gauge couplings, $r \equiv g_{5L}/g_{5R}$, three parameters remain to be determined by the matching procedure, namely g_{5L} , \tilde{g}_5 and v_H . To fit them we proceed as follows. First, we relate \tilde{g}_5 and g_{5L} by imposing that the (canonically normalized) photon couples to the electric charge $Q = T_{3,L} + T_{3,R} - Q_{B-L}/2$. Given the KK decomposition above we get:

$$\frac{1}{e^2} = \left(\frac{1+r^2}{g_{5L}^2} + \frac{1}{\tilde{g}_5^2} \right) R \log(R'/R). \quad (C.6)$$

Then, the charged boson quantization equation fixes the product $g_{5L}v_H$ as a function of m_W . And plugging back these two relations into the neutral boson quantization equation

⁵This can be thought as being the result of a Higgs mechanism in the limit where the localized scalar is decoupled.

allows to find the Higgs VEV as a function of m_Z , m_W and e . Finally to fully determine the wave-functions we need to make the W and the Z are canonical fields by imposing:

$$\int_R^R dz \left(\frac{R}{z} \right) [a^{L,\pm}(z, m_W)^2 + a^{R,\pm}(z, m_W)^2] = 1 \quad (\text{C.7})$$

$$\int_R^R dz \left(\frac{R}{z} \right) [a^{L,3}(z, m_Z)^2 + a^{R,3}(z, m_Z)^2 + a^X(z, m_Z)^2] = 1. \quad (\text{C.8})$$

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